Mathematica 11.3 Integration Test Results

Test results for the 201 problems in "6.5.3 Hyperbolic secant functions.m"

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \sqrt{\text{Sech}[a+b\,x]^2}\,dx$$

Optimal (type 3, 11 leaves, 2 steps):

$$\frac{\text{ArcSin}[\text{Tanh}[a+b\,x]]}{b}$$

Result (type 3, 34 leaves):

$$\frac{2\,\text{ArcTan}\left[\text{Tanh}\left[\frac{1}{2}\,(a+b\,x)\right]\right]\,\text{Cosh}[a+b\,x]\,\sqrt{\text{Sech}[a+b\,x]^2}}{b}$$

Problem 79: Result more than twice size of optimal antiderivative.

$$\int (a+a\,\text{Sech}[c+d\,x])^{3/2}\,dx$$

Optimal (type 3, 66 leaves, 4 steps):

$$\frac{2\,a^{3/2}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Tanh}[c+d\,x]}{\sqrt{a+a\,\text{Sech}[c+d\,x]}}\right]}{d}+\frac{2\,a^2\,\text{Tanh}[c+d\,x]}{d\,\sqrt{a+a\,\text{Sech}[c+d\,x]}}$$

Result (type 3, 135 leaves):

$$\frac{1}{d\,\left(1+e^{c+d\,x}\right)}a\,\left(-2+2\,e^{c+d\,x}+c\,\sqrt{1+e^{2\,(c+d\,x)}}+d\,\sqrt{1+e^{2\,(c+d\,x)}}\,x+\sqrt{1+e^{2\,(c+d\,x)}}\,\text{ArcSinh}\left[e^{c+d\,x}\right]-\sqrt{1+e^{2\,(c+d\,x)}}\,\text{Log}\left[1+\sqrt{1+e^{2\,(c+d\,x)}}\right]\right)\sqrt{a\,(1+\text{Sech}[c+d\,x])}$$

Problem 80: Result more than twice size of optimal antiderivative.

$$\int \sqrt{a+a\,\text{Sech}[c+d\,x]}\,dx$$

Optimal (type 3, 37 leaves, 2 steps):

$$\frac{2\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Tanh}[c+d\,x]}{\sqrt{a+a\,\text{Sech}[c+d\,x]}}\right]}{d}$$

Result (type 3, 77 leaves):

$$\frac{1}{d\left(1+e^{c+d\,x}\right)}\sqrt{1+e^{2\,(c+d\,x)}}\left(c+d\,x+\text{ArcSinh}\left[e^{c+d\,x}\right]-\text{Log}\left[1+\sqrt{1+e^{2\,(c+d\,x)}}\right]\right)\sqrt{a\left(1+\text{Sech}[c+d\,x]\right)}$$

Problem 82: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+a\,\text{Sech}[c+d\,x]\right)^{3/2}}\,dx$$

Optimal (type 3, 114 leaves, 6 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Tanh}[c+d\,x]}{\sqrt{a+a\,\text{Sech}[c+d\,x]}}\right]}{a^{3/2}\,d}-\frac{5\,\text{ArcTanh}\left[\frac{\sqrt{a}\,\text{Tanh}[c+d\,x]}{\sqrt{2}\,\sqrt{a+a\,\text{Sech}[c+d\,x]}}\right]}{2\sqrt{2}\,a^{3/2}\,d}-\frac{\text{Tanh}[c+d\,x]}{2\,d\left(a+a\,\text{Sech}[c+d\,x]\right)^{3/2}}$$

Result (type 3, 231 leaves):

$$\left(\text{Cosh}\left[\frac{1}{2}(c+d\,x)\right]^3\text{Sech}[c+d\,x]^{3/2}\left(\sqrt{2}\,e^{\frac{1}{2}(-c-d\,x)}\sqrt{\frac{e^{c+d\,x}}{1+e^{2\,(c+d\,x)}}}\sqrt{1+e^{2\,(c+d\,x)}}\left(4\,c+4\,d\,x+4\,\text{ArcSinh}\left[e^{c+d\,x}\right]-5\sqrt{2}\,\text{Log}\left[1+e^{c+d\,x}\right]-4\,\text{Log}\left[1+\sqrt{1+e^{2\,(c+d\,x)}}\right]+5\sqrt{2}\,\text{Log}\left[1-e^{c+d\,x}+\sqrt{2}\sqrt{1+e^{2\,(c+d\,x)}}\right]\right)-\frac{2\,\text{Sech}\left[\frac{1}{2}(c+d\,x)\right]\text{Tanh}\left[\frac{1}{2}(c+d\,x)\right]}{\sqrt{\text{Sech}[c+d\,x]}}\right)\right)\Big/\left(2\,d\left(a\left(1+\text{Sech}[c+d\,x]\right)\right)^{3/2}\right)$$

Problem 85: Result more than twice size of optimal antiderivative.

$$\int\sqrt{3+3\,\text{Sech}[x]}\,dx$$

Optimal (type 3, 19 leaves, 2 steps):

$$2\sqrt{3}\,\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1+\text{Sech}[x]}}\right]$$

Result (type 3, 54 leaves):

$$\frac{\sqrt{3}\sqrt{1+e^{2\,x}}\left(x+\text{ArcSinh}\left[e^x\right]-\text{Log}\left[1+\sqrt{1+e^{2\,x}}\right]\right)\sqrt{1+\text{Sech}[x]}}{1+e^x}$$

Problem 86: Result more than twice size of optimal antiderivative.

$$\int \sqrt{3-3\,\text{Sech}[x]}\,dx$$

Optimal (type 3, 21 leaves, 2 steps):

$$2\sqrt{3}\,\text{ArcTanh}\left[\frac{\text{Tanh}[x]}{\sqrt{1-\text{Sech}[x]}}\right]$$

Result (type 3, 56 leaves):

$$\frac{\sqrt{3}\,\sqrt{1+e^{2x}}\,\left(-x+\text{ArcSinh}\left[e^{x}\right]+\text{Log}\left[1+\sqrt{1+e^{2x}}\right]\right)\sqrt{1-\text{Sech}[x]}}{-1+e^{x}}$$

Problem 94: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b\,\text{Sech}[c+d\,x]}}\,dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{1}{a\,d}2\sqrt{a+b}\,\text{Coth}[c+d\,x]\,\text{EllipticPi}\left[\frac{a+b}{a},\,\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\,\frac{a+b}{a-b}\right]$$
$$\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a+b\,\text{Sech}[c+d\,x]}}\,dx$$

Problem 129: Result more than twice size of optimal antiderivative.

$$\int \text{Coth}[c+d\,x]^3\,\sqrt{a+b\,\text{Sech}[c+d\,x]}\,dx$$

Optimal (type 3, 217 leaves, 13 steps):

$$\frac{2\sqrt{a}\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a}}\right]}{d}-\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}\,d}+\frac{3\,b\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a-b}}\right]}{4\sqrt{a-b}\,d}-$$
$$\frac{a\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b}\,d}-\frac{3\,b\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right]}{4\sqrt{a+b}\,d}-\frac{\text{Coth}[c+d\,x]^2\,\sqrt{a+b\,\text{Sech}[c+d\,x]}}{2\,d}$$

Result (type 3, 844 leaves):

$$\begin{aligned}
&\frac{\left(-\frac{1}{2}-\frac{1}{2}\text{Csch}[c+d x]^2\right)\sqrt{a+b\,\text{Sech}[c+d x]}}{d}+\frac{1}{4 d\sqrt{b+a\,\text{Cosh}[c+d x]}\sqrt{\text{Sech}[c+d x]}} \\
&\Bigg(\Bigg(3 b\left(\sqrt{a-b}\,\text{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{-a-b}\sqrt{a\,\text{Cosh}[c+d x]}}\right]+\sqrt{-a-b}\right. \\
&\left.\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{a-b}\sqrt{a\,\text{Cosh}[c+d x]}}\right]\right)\sqrt{\frac{-a+a\,\text{Cosh}[c+d x]}{a+a\,\text{Cosh}[c+d x]}}\left(a+a\,\text{Cosh}[c+d x]\right)\Bigg)\Big/ \\
&\Big(\sqrt{a}\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\text{Cosh}[c+d x]}\sqrt{a\,\text{Cosh}[c+d x]}\sqrt{1+\text{Cosh}[c+d x]} \\
&\sqrt{\text{Sech}[c+d x]}\Big)+\Bigg(2\left(\sqrt{a-b}\,\text{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{-a-b}\sqrt{a\,\text{Cosh}[c+d x]}}\right]-\right. \\
&\left.\sqrt{-a-b}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{a-b}\sqrt{a\,\text{Cosh}[c+d x]}}\right]\right)\sqrt{a\,\text{Cosh}[c+d x]} \\
&\sqrt{\frac{-a+a\,\text{Cosh}[c+d x]}{a+a\,\text{Cosh}[c+d x]}}\left(a+a\,\text{Cosh}[c+d x]\right)\sqrt{\text{Sech}[c+d x]}\Bigg)\Big/ \\
&\left(\sqrt{a}\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\text{Cosh}[c+d x]}\sqrt{1+\text{Cosh}[c+d x]}\right)+\Bigg(2 a\Bigg(\sqrt{-a-b} \\
&\left(-4\sqrt{a-b}\,\text{ArcTan}\left[\frac{\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{-a\,\text{Cosh}[c+d x]}}\right]+\sqrt{a}\,\text{ArcTan}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{a-b}\sqrt{-a\,\text{Cosh}[c+d x]}}\right]\right)- \\
&\sqrt{a}\sqrt{a-b}\,\text{ArcTanh}\left[\frac{\sqrt{a}\sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{-a-b}\sqrt{-a\,\text{Cosh}[c+d x]}}\right]\Bigg)\sqrt{-a\,\text{Cosh}[c+d x]} \\
&\sqrt{\frac{-a+a\,\text{Cosh}[c+d x]}{a+a\,\text{Cosh}[c+d x]}}\left(a+a\,\text{Cosh}[c+d x]\right)\text{Cosh}[2(c+d x)]\sqrt{\text{Sech}[c+d x]}\Bigg)\Big/ \\
&\Big(\sqrt{-a-b}\sqrt{a-b}\sqrt{-1+\text{Cosh}[c+d x]}\sqrt{1+\text{Cosh}[c+d x]} \\
&\left(a^2-2 b^2+4 b\left(b+a\,\text{Cosh}[c+d x]\right)-2\left(b+a\,\text{Cosh}[c+d x]\right)^2\right)\Big)\Bigg)\sqrt{a+b\,\text{Sech}[c+d x]}
\end{aligned}$$

Problem 130: Attempted integration timed out after 120 seconds.

$$\int\sqrt{a+b\,\text{Sech}[c+d x]}\,\text{Tanh}[c+d x]^2\,dx$$

Optimal (type 4, 344 leaves, 7 steps):

$$-\frac{1}{3 b^2 d}2 a (a-b) \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} - \frac{1}{3 b d}$$

$$2 \sqrt{a+b} (a+2 b) \text{Coth}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{d}$$

$$2 \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} - \frac{2 \sqrt{a+b \text{Sech}[c+d x]} \text{Tanh}[c+d x]}{3 d}$$

Result (type 1, 1 leaves):

???

Problem 131: Unable to integrate problem.

$$\int \sqrt{a+b \text{Sech}[c+d x]} \, dx$$

Optimal (type 4, 125 leaves, 1 step):

$$\frac{1}{\sqrt{a+b} d}2 \text{Coth}[c+d x] \text{EllipticPi}\left[\frac{a}{a+b}, \text{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b \text{Sech}[c+d x]}}\right], \frac{a-b}{a+b}\right] \sqrt{-\frac{b (1-\text{Sech}[c+d x])}{a+b \text{Sech}[c+d x]}} \sqrt{\frac{b (1+\text{Sech}[c+d x])}{a+b \text{Sech}[c+d x]}} (a+b \text{Sech}[c+d x])$$

Result (type 8, 16 leaves):

$$\int \sqrt{a+b \text{Sech}[c+d x]} \, dx$$

Problem 132: Attempted integration timed out after 120 seconds.

$$\int \text{Coth}[c+d x]^2 \sqrt{a+b \text{Sech}[c+d x]} \, dx$$

Optimal (type 4, 246 leaves, 5 steps):

$$\frac{1}{d}\sqrt{a+b}\ \text{Coth}[c+d\,x]\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-\frac{\text{Coth}[c+d\,x]\sqrt{a+b\,\text{Sech}[c+d\,x]}}{d}+$$

$$\frac{1}{\sqrt{a+b}\ d}2\,\text{Coth}[c+d\,x]\ \text{EllipticPi}\left[\frac{a}{a+b},\text{ArcSin}\left[\frac{\sqrt{a+b}}{\sqrt{a+b\,\text{Sech}[c+d\,x]}}\right],\frac{a-b}{a+b}\right]\sqrt{-\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b\,\text{Sech}[c+d\,x]}}\sqrt{\frac{b\,(1+\text{Sech}[c+d\,x])}{a+b\,\text{Sech}[c+d\,x]}}\ (a+b\,\text{Sech}[c+d\,x])$$

Result (type 1, 1 leaves):

???

Problem 135: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Tanh}[c+d\,x]}{\sqrt{a+b\,\text{Sech}[c+d\,x]}}\,dx$$

Optimal (type 3, 31 leaves, 3 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a}}\right]}{\sqrt{a}\ d}$$

Result (type 3, 82 leaves):

$$\left(2\sqrt{b+a\,\text{Cosh}[c+d\,x]}\ \text{Log}\left[a\sqrt{b+a\,\text{Cosh}[c+d\,x]}+\frac{a^{3/2}}{\sqrt{\text{Sech}[c+d\,x]}}\right]\sqrt{\text{Sech}[c+d\,x]}\right)\Big/\left(\sqrt{a}\ d\sqrt{a+b\,\text{Sech}[c+d\,x]}\right)$$

Problem 136: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[c+d\,x]}{\sqrt{a+b\,\text{Sech}[c+d\,x]}}\,dx$$

Optimal (type 3, 106 leaves, 7 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a}}\right]}{\sqrt{a}\ d}-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}\ d}-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b}\ d}$$

Result (type 3, 419 leaves):

$$\frac{1}{2\,a\,\sqrt{-a-b}\,\sqrt{a-b}\,d\,\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}$$

$$\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}\,\left(4\,\sqrt{-a-b}\,\sqrt{a-b}\,\mathrm{ArcTan}\left[\frac{\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}}{\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}}\right]\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}\;-\right.$$

$$\sqrt{a}\,\sqrt{-a-b}\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}}\right]\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}\;+$$

$$\sqrt{a}\,\sqrt{a-b}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}}\right]\sqrt{-a\,\mathrm{Cosh}[c+d\,x]}\;+$$

$$\sqrt{a}\,\sqrt{a-b}\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\,\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{a\,\mathrm{Cosh}[c+d\,x]}}\right]\sqrt{a\,\mathrm{Cosh}[c+d\,x]}\;-$$

$$\left.\sqrt{a}\,\sqrt{-a-b}\,\mathrm{ArcTanh}\left[\frac{\sqrt{a}\,\sqrt{b+a\,\mathrm{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{a\,\mathrm{Cosh}[c+d\,x]}}\right]\sqrt{a\,\mathrm{Cosh}[c+d\,x]}\right)\mathrm{Sech}[c+d\,x]$$

Problem 137: Result more than twice size of optimal antiderivative.

$$\int \frac{\mathrm{Coth}[c+d\,x]^3}{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}\,dx$$

Optimal (type 3, 262 leaves, 11 steps):

$$\frac{2\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{\sqrt{a}}\right]}{\sqrt{a}\,d}-\frac{\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{\sqrt{a-b}}\right]}{\sqrt{a-b}\,d}+\frac{b\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{\sqrt{a-b}}\right]}{4\,(a-b)^{3/2}\,d}-\frac{b\,\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{\sqrt{a+b}}\right]}{4\,(a+b)^{3/2}\,d}-\frac{\mathrm{ArcTanh}\left[\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{\sqrt{a+b}}\right]}{\sqrt{a+b}\,d}-\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{4\,(a+b)\,d\,(1-\mathrm{Sech}[c+d\,x])}-\frac{\sqrt{a+b\,\mathrm{Sech}[c+d\,x]}}{4\,(a-b)\,d\,(1+\mathrm{Sech}[c+d\,x])}$$

Result (type 3, 925 leaves):

$$
\begin{aligned}
&\frac{1}{4\,(a-b)\,(a+b)\,d\,\sqrt{a+b\,\text{Sech}[c+d\,x]}}\\
&\sqrt{b+a\,\text{Cosh}[c+d\,x]}\,\Bigg(\Bigg(\sqrt{a}\,b\,\Bigg(\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]+\sqrt{-a-b}\\
&\quad\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\,\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,\big(a+a\,\text{Cosh}[c+d\,x]\big)\Bigg)\Big/\\
&\Big(\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{a\,\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\\
&\quad\sqrt{\text{Sech}[c+d\,x]}\Big)+\Bigg(\big(2\,a^2-3\,b^2\big)\,\Bigg(\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]-\\
&\quad\sqrt{-a-b}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\,\sqrt{a\,\text{Cosh}[c+d\,x]}\\
&\quad\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,\big(a+a\,\text{Cosh}[c+d\,x]\big)\,\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Big/\\
&\Big(a^{3/2}\,\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\Big)+\Bigg(\big(2\,a^2-2\,b^2\big)\,\Bigg(\sqrt{-a-b}\\
&\quad\Bigg(-4\,\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]+\sqrt{a}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)-\\
&\quad\sqrt{a}\,\sqrt{a-b}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\,\sqrt{-a\,\text{Cosh}[c+d\,x]}\\
&\quad\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,\big(a+a\,\text{Cosh}[c+d\,x]\big)\,\text{Cosh}\big[2\,(c+d\,x)\big]\,\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Big/\\
&\Big(\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\\
&\quad\big(a^2-2\,b^2+4\,b\,\big(b+a\,\text{Cosh}[c+d\,x]\big)-2\,\big(b+a\,\text{Cosh}[c+d\,x]\big)^2\big)\Big)\Bigg)\,\sqrt{\text{Sech}[c+d\,x]}+\\
&\Bigg(\big(b+a\,\text{Cosh}[c+d\,x]\big)\,\Bigg(-\frac{a}{2\,\big(a^2-b^2\big)}+\frac{\big(a-b\,\text{Cosh}[c+d\,x]\big)\,\text{Csch}[c+d\,x]^2}{2\,\big(-a^2+b^2\big)}\Bigg)\\
&\quad\text{Sech}[c+d\,x]\Bigg)\Big/\Big(d\\
&\quad\sqrt{a+b\,\text{Sech}[c+d\,x]}\Big)
\end{aligned}
$$

Problem 138: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[c+d x]^4}{\sqrt{a+b\,\text{Sech}[c+d x]}}\,dx$$

Optimal (type 4, 610 leaves, 11 steps):

$$-\frac{1}{b^2 d}4 (a-b) \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{15 b^4 d}$$

$$2 (a-b) \sqrt{a+b}\ \left(8 a^2+9 b^2\right) \text{Coth}[c+d x]\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} - \frac{1}{b d}$$

$$4 \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{15 b^3 d}$$

$$2 \sqrt{a+b}\ \left(8 a^2-2 a b+9 b^2\right) \text{Coth}[c+d x]\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{a d}$$

$$2 \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} -$$

$$\frac{8 a \sqrt{a+b\,\text{Sech}[c+d x]}\ \text{Tanh}[c+d x]}{15 b^2 d} + \frac{2\,\text{Sech}[c+d x] \sqrt{a+b\,\text{Sech}[c+d x]}\ \text{Tanh}[c+d x]}{5 b d}$$

Result (type 1, 1 leaves):

???

Problem 139: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[c+d x]^2}{\sqrt{a+b\,\text{Sech}[c+d x]}}\,dx$$

Optimal (type 4, 310 leaves, 6 steps):

$$-\frac{1}{b^2 d}2 (a-b) \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} - \frac{1}{b d} 2 \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{a d} 2 \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}}$$

Result (type 1, 1 leaves):

???

Problem 140: Unable to integrate problem.

$$\int \frac{1}{\sqrt{a+b \text{Sech}[c+d x]}} \, dx$$

Optimal (type 4, 106 leaves, 1 step):

$$\frac{1}{a d}2 \sqrt{a+b} \text{Coth}[c+d x] \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b \text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}}$$

Result (type 8, 16 leaves):

$$\int \frac{1}{\sqrt{a+b \text{Sech}[c+d x]}} \, dx$$

Problem 141: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Coth}[c+d x]^2}{\sqrt{a+b \text{Sech}[c+d x]}} \, dx$$

Optimal (type 4, 362 leaves, 9 steps):

$$\frac{1}{\sqrt{a+b}\ d}\text{Coth}[c+d x]\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b\,(1-\text{Sech}[c+d x])}{a+b}}\ \sqrt{-\frac{b\,(1+\text{Sech}[c+d x])}{a-b}}-\frac{1}{\sqrt{a+b}\ d}$$

$$\text{Coth}[c+d x]\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b\,(1-\text{Sech}[c+d x])}{a+b}}\ \sqrt{-\frac{b\,(1+\text{Sech}[c+d x])}{a-b}}+\frac{1}{a\,d}$$

$$2\sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticPi}\left[\frac{a+b}{a},\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$

$$\sqrt{\frac{b\,(1-\text{Sech}[c+d x])}{a+b}}\ \sqrt{-\frac{b\,(1+\text{Sech}[c+d x])}{a-b}}-$$

$$\frac{\text{Coth}[c+d x]}{d\sqrt{a+b\,\text{Sech}[c+d x]}}-\frac{b^2\,\text{Tanh}[c+d x]}{(a^2-b^2)\ d\sqrt{a+b\,\text{Sech}[c+d x]}}$$

Result (type 1, 1 leaves):

???

Problem 145: Result more than twice size of optimal antiderivative.

$$\int\frac{\text{Coth}[c+d x]}{(a+b\,\text{Sech}[c+d x])^{3/2}}\,dx$$

Optimal (type 3, 142 leaves, 7 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2}\,d}-\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{(a-b)^{3/2}\,d}-$$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{(a+b)^{3/2}\,d}+\frac{2\,b^2}{a\,(a^2-b^2)\ d\sqrt{a+b\,\text{Sech}[c+d x]}}$$

Result (type 3, 927 leaves):

$$-\frac{1}{2\,a\,(-a+b)\,(a+b)\,d\,(a+b\,\text{Sech}[c+d x])^{3/2}}$$

$$(b+a\,\text{Cosh}[c+d x])^{3/2}\left(-\left(\left(2\sqrt{a}\ b\left(\sqrt{a-b}\ \text{ArcTan}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d x]}}{\sqrt{-a-b}\ \sqrt{a\,\text{Cosh}[c+d x]}}\right]+\right.\right.\right.\right.$$

$$\sqrt{-a-b}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\ \sqrt{a\,\text{Cosh}[c+d\,x]}}\right]\right)\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}$$

$$\left(a+a\,\text{Cosh}[c+d\,x]\right)\Bigg)\Bigg/\left(\sqrt{-a-b}\ \sqrt{a-b}\ \sqrt{-1+\text{Cosh}[c+d\,x]}\right.$$

$$\left.\sqrt{a\,\text{Cosh}[c+d\,x]}\ \sqrt{1+\text{Cosh}[c+d\,x]}\ \sqrt{\text{Sech}[c+d\,x]}\right)\Bigg)+$$

$$\Bigg(\left(a^2+b^2\right)\left(\sqrt{a-b}\ \text{ArcTan}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\ \sqrt{a\,\text{Cosh}[c+d\,x]}}\right]-\right.$$

$$\left.\sqrt{-a-b}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\ \sqrt{a\,\text{Cosh}[c+d\,x]}}\right]\right)\sqrt{a\,\text{Cosh}[c+d\,x]}$$

$$\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\ \left(a+a\,\text{Cosh}[c+d\,x]\right)\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Bigg/$$

$$\left(a^{3/2}\sqrt{-a-b}\ \sqrt{a-b}\ \sqrt{-1+\text{Cosh}[c+d\,x]}\ \sqrt{1+\text{Cosh}[c+d\,x]}\right)+$$

$$\Bigg(\left(a^2-b^2\right)\left(\sqrt{-a-b}\ \left(-4\sqrt{a-b}\ \text{ArcTan}\left[\frac{\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a\,\text{Cosh}[c+d\,x]}}\right]+\right.\right.$$

$$\left.\sqrt{a}\ \text{ArcTan}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\ \sqrt{-a\,\text{Cosh}[c+d\,x]}}\right]\right)-$$

$$\left.\sqrt{a}\ \sqrt{a-b}\ \text{ArcTanh}\left[\frac{\sqrt{a}\ \sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\ \sqrt{-a\,\text{Cosh}[c+d\,x]}}\right]\right)\sqrt{-a\,\text{Cosh}[c+d\,x]}$$

$$\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\ \left(a+a\,\text{Cosh}[c+d\,x]\right)\text{Cosh}\left[2\left(c+d\,x\right)\right]\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Bigg/$$

$$\left(\sqrt{-a-b}\ \sqrt{a-b}\ \sqrt{-1+\text{Cosh}[c+d\,x]}\ \sqrt{1+\text{Cosh}[c+d\,x]}\right.$$

$$\left.\left(a^2-2\,b^2+4\,b\left(b+a\,\text{Cosh}[c+d\,x]\right)-2\left(b+a\,\text{Cosh}[c+d\,x]\right)^2\right)\right)\Bigg)\text{Sech}[c+d\,x]^{3/2}+$$

$$\left(\left(b+a\,\text{Cosh}[c+d\,x]\right)^2\left(-\frac{2\,b^2}{a^2\left(-a^2+b^2\right)}-\frac{2\,b^3}{a^2\left(a^2-b^2\right)\left(b+a\,\text{Cosh}[c+d\,x]\right)}\right)\right.$$

$$\left.\text{Sech}[c+d\,x]^2\right)\Bigg/\left(d\right.$$

$$\left.\left(a+b\,\text{Sech}[c+d\,x]\right)^{3/2}\right)$$

Problem 146: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{Coth}[c+d x]^3}{(a+b\,\text{Sech}[c+d x])^{3/2}}\,dx$$

Optimal (type 3, 316 leaves, 11 steps):

$$\frac{2\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a}}\right]}{a^{3/2}\,d}-\frac{(2 a-3 b)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{2\,(a-b)^{5/2}\,d}+$$

$$\frac{b\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a-b}}\right]}{4\,(a-b)^{5/2}\,d}-\frac{b\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{4\,(a+b)^{5/2}\,d}-\frac{(2 a+3 b)\,\text{ArcTanh}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right]}{2\,(a+b)^{5/2}\,d}-$$

$$\frac{2\,b^4}{a\,(a^2-b^2)^2\,d\,\sqrt{a+b\,\text{Sech}[c+d x]}}-\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{4\,(a+b)^2\,d\,(1-\text{Sech}[c+d x])}-\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{4\,(a-b)^2\,d\,(1+\text{Sech}[c+d x])}$$

Result (type 3, 1019 leaves):

$$\frac{1}{4\,a\,(a-b)^2\,(a+b)^2\,d\,(a+b\,\text{Sech}[c+d\,x])^{3/2}}(b+a\,\text{Cosh}[c+d\,x])^{3/2}\Bigg(\Bigg(\Bigg((-a^3 b+7 a b^3)\Bigg(\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]+\sqrt{-a-b}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,(a+a\,\text{Cosh}[c+d\,x])\Bigg)\Big/\Big(\sqrt{a}\,\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{a\,\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\,\sqrt{\text{Sech}[c+d\,x]}\Big)+\Bigg((2 a^4-6 a^2 b^2-2 b^4)\Bigg(\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]-\sqrt{-a-b}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\sqrt{a\,\text{Cosh}[c+d\,x]}\,\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,(a+a\,\text{Cosh}[c+d\,x])\,\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Big/\Big(a^{3/2}\,\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\Big)+\Bigg((2 a^4-4 a^2 b^2+2 b^4)\Bigg(\sqrt{-a-b}\Bigg(-4\sqrt{a-b}\,\text{ArcTan}\Big[\frac{\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]+\sqrt{a}\,\text{ArcTan}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{a-b}\,\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)-\sqrt{a}\,\sqrt{a-b}\,\text{ArcTanh}\Big[\frac{\sqrt{a}\,\sqrt{b+a\,\text{Cosh}[c+d\,x]}}{\sqrt{-a-b}\,\sqrt{-a\,\text{Cosh}[c+d\,x]}}\Big]\Bigg)\sqrt{-a\,\text{Cosh}[c+d\,x]}\,\sqrt{\frac{-a+a\,\text{Cosh}[c+d\,x]}{a+a\,\text{Cosh}[c+d\,x]}}\,(a+a\,\text{Cosh}[c+d\,x])\,\text{Cosh}[2\,(c+d\,x)]\,\sqrt{\text{Sech}[c+d\,x]}\Bigg)\Big/\Big(\sqrt{-a-b}\,\sqrt{a-b}\,\sqrt{-1+\text{Cosh}[c+d\,x]}\,\sqrt{1+\text{Cosh}[c+d\,x]}\,\big(a^2-2 b^2+4 b\,(b+a\,\text{Cosh}[c+d\,x])-2\,(b+a\,\text{Cosh}[c+d\,x])^2\big)\Big)\Bigg)\,\text{Sech}[c+d\,x]^{3/2}+$$

$$\Bigg((b+a\,\text{Cosh}[c+d\,x])^2\Bigg(-\frac{a^4+a^2 b^2+4 b^4}{2 a^2\,(-a^2+b^2)^2}+\frac{2 b^5}{a^2\,(a^2-b^2)^2\,(b+a\,\text{Cosh}[c+d\,x])}+\frac{(-a^2-b^2+2 a b\,\text{Cosh}[c+d\,x])\,\text{Csch}[c+d\,x]^2}{2\,(-a^2+b^2)^2}\Bigg)\text{Sech}[c+d\,x]^2\Bigg)\Big/\Big(d\,(a+b\,\text{Sech}[c+d\,x])^{3/2}\Big)$$

Problem 147: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[c + d x]^4}{\left(a + b\,\text{Sech}[c + d x]\right)^{3/2}}\, dx$$

Optimal (type 4, 907 leaves, 17 steps) :

$$
-\frac{1}{a\sqrt{a+b}\,d}2\,\text{Coth}[c+d\,x]\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{b^2\sqrt{a+b}\,d}
4\,a\,\text{Coth}[c+d\,x]\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-\frac{1}{3\,b^4\sqrt{a+b}\,d}
2\,a\,(8\,a^2-5\,b^2)\,\text{Coth}[c+d\,x]\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{a\sqrt{a+b}\,d}
2\,\text{Coth}[c+d\,x]\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{b\sqrt{a+b}\,d}
4\,\text{Coth}[c+d\,x]\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-\frac{1}{3\,b^3\sqrt{a+b}\,d}
2\,(2\,a+b)\,(4\,a+b)\,\text{Coth}[c+d\,x]\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{a^2\,d}
2\sqrt{a+b}\,\text{Coth}[c+d\,x]\,\text{EllipticPi}\left[\frac{a+b}{a},\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]
\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-\frac{4\,a\,\text{Tanh}[c+d\,x]}{(a^2-b^2)\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}+
\frac{2\,b^2\,\text{Tanh}[c+d\,x]}{a\,(a^2-b^2)\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}-\frac{2\,a^2\,\text{Sech}[c+d\,x]\,\text{Tanh}[c+d\,x]}{b\,(a^2-b^2)\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}+
\frac{2\,(4\,a^2-b^2)\sqrt{a+b\,\text{Sech}[c+d\,x]}\,\text{Tanh}[c+d\,x]}{3\,b^2\,(a^2-b^2)\,d}
$$

Result (type 1, 1 leaves):

???

Problem 148: Attempted integration timed out after 120 seconds.

$$\int \frac{\text{Tanh}[c+d x]^2}{(a+b\,\text{Sech}[c+d x])^{3/2}}\,dx$$

Optimal (type 4, 344 leaves, 7 steps):

$$\frac{1}{a b^2 d}2 (a-b) \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{a b d} 2 \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} + \frac{1}{a^2 d} 2 \sqrt{a+b}\ \text{Coth}[c+d x]\ \text{EllipticPi}\left[\frac{a+b}{a}, \text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d x]}}{\sqrt{a+b}}\right], \frac{a+b}{a-b}\right] \sqrt{\frac{b (1-\text{Sech}[c+d x])}{a+b}} \sqrt{-\frac{b (1+\text{Sech}[c+d x])}{a-b}} - \frac{2\,\text{Tanh}[c+d x]}{a d \sqrt{a+b\,\text{Sech}[c+d x]}}$$

Result (type 1, 1 leaves):

???

Problem 149: Attempted integration timed out after 120 seconds.

$$\int \frac{1}{(a+b\,\text{Sech}[c+d x])^{3/2}}\,dx$$

Optimal (type 4, 347 leaves, 6 steps):

$$-\frac{1}{a\sqrt{a+b}\,d}2\,\text{Coth}[c+d\,x]\,\text{EllipticE}\Big[\text{ArcSin}\Big[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\Big],\frac{a+b}{a-b}\Big]\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{a\sqrt{a+b}\,d}$$

$$2\,\text{Coth}[c+d\,x]\,\text{EllipticF}\Big[\text{ArcSin}\Big[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\Big],\frac{a+b}{a-b}\Big]\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{a^2\,d}$$

$$2\sqrt{a+b}\,\text{Coth}[c+d\,x]\,\text{EllipticPi}\Big[\frac{a+b}{a},\text{ArcSin}\Big[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\Big],\frac{a+b}{a-b}\Big]$$

$$\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{2\,b^2\,\text{Tanh}[c+d\,x]}{a\,(a^2-b^2)\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}$$

Result (type 1, 1 leaves):

???

Problem 150: Attempted integration timed out after 120 seconds.

$$\int\frac{\text{Coth}[c+d\,x]^2}{(a+b\,\text{Sech}[c+d\,x])^{3/2}}\,dx$$

Optimal (type 4, 665 leaves, 14 steps):

$$\left(4\,a\,\text{Coth}[c+d\,x]\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]\right.$$
$$\left.\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}\right)\Big/\left((a-b)\,(a+b)^{3/2}\,d\right)-$$
$$\frac{1}{a\sqrt{a+b}\,d}2\,\text{Coth}[c+d\,x]\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$
$$\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-$$
$$\left((3\,a-b)\,\text{Coth}[c+d\,x]\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]\right.$$
$$\left.\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}\right)\Big/\left((a-b)\,(a+b)^{3/2}\,d\right)+$$
$$\frac{1}{a\sqrt{a+b}\,d}2\,\text{Coth}[c+d\,x]\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$
$$\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}+\frac{1}{a^2\,d}$$
$$2\sqrt{a+b}\,\text{Coth}[c+d\,x]\,\text{EllipticPi}\left[\frac{a+b}{a},\text{ArcSin}\left[\frac{\sqrt{a+b\,\text{Sech}[c+d\,x]}}{\sqrt{a+b}}\right],\frac{a+b}{a-b}\right]$$
$$\sqrt{\frac{b\,(1-\text{Sech}[c+d\,x])}{a+b}}\,\sqrt{-\frac{b\,(1+\text{Sech}[c+d\,x])}{a-b}}-$$
$$\frac{\text{Coth}[c+d\,x]}{d\,(a+b\,\text{Sech}[c+d\,x])^{3/2}}-\frac{b^2\,\text{Tanh}[c+d\,x]}{(a^2-b^2)\,d\,(a+b\,\text{Sech}[c+d\,x])^{3/2}}-$$
$$\frac{4\,a\,b^2\,\text{Tanh}[c+d\,x]}{(a^2-b^2)^2\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}+\frac{2\,b^2\,\text{Tanh}[c+d\,x]}{a\,(a^2-b^2)\,d\sqrt{a+b\,\text{Sech}[c+d\,x]}}$$

Result (type 1, 1 leaves):

???

Problem 158: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^5}{\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}\,dx$$

Optimal (type 4, 108 leaves, 6 steps):

$$\frac{2 x^2}{21 c^4 \sqrt{\text{Sech}[2 \text{Log}[c x]]}}+\frac{x^6}{7 \sqrt{\text{Sech}[2 \text{Log}[c x]]}}+\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}} \left(c^2+\frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}[c x],\frac{1}{2}\right]}{21 c^5 \left(c^4+\frac{1}{x^4}\right) x \sqrt{\text{Sech}[2 \text{Log}[c x]]}}$$

Result (type 4, 120 leaves):

$$\frac{1}{21 \sqrt{2} c^6 \sqrt{c^2 x^2}} \sqrt{\frac{c^2 x^2}{1+c^4 x^4}} \left(\sqrt{c^2 x^2} \left(2+5 c^4 x^4+3 c^8 x^8\right)+2 (-1)^{1/4} \sqrt{1+c^4 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right],-1\right]\right)$$

Problem 160: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3}{\sqrt{\text{Sech}[2 \text{Log}[c x]]}} \, dx$$

Optimal (type 4, 203 leaves, 8 steps):

$$\frac{2}{5 c^4 \sqrt{\text{Sech}[2 \text{Log}[c x]]}}-\frac{2}{5 c^4 \left(c^2+\frac{1}{x^2}\right) x^2 \sqrt{\text{Sech}[2 \text{Log}[c x]]}}+\frac{x^4}{5 \sqrt{\text{Sech}[2 \text{Log}[c x]]}}+\frac{2 \sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}} \left(c^2+\frac{1}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}[c x],\frac{1}{2}\right]}{5 c^3 \left(c^4+\frac{1}{x^4}\right) x \sqrt{\text{Sech}[2 \text{Log}[c x]]}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}} \left(c^2+\frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}[c x],\frac{1}{2}\right]}{5 c^3 \left(c^4+\frac{1}{x^4}\right) x \sqrt{\text{Sech}[2 \text{Log}[c x]]}}$$

Result (type 4, 155 leaves):

$$\frac{1}{5 \sqrt{2} c^4 \sqrt{c^2 x^2}} \sqrt{\frac{c^2 x^2}{1+c^4 x^4}} \left(\left(c^2 x^2\right)^{3/2} \left(1+c^4 x^4\right)-2 (-1)^{3/4} \sqrt{1+c^4 x^4} \text{EllipticE}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right],-1\right]+2 (-1)^{3/4} \sqrt{1+c^4 x^4} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right],-1\right]\right)$$

Problem 162: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}\,dx$$

Optimal (type 4, 87 leaves, 5 steps):

$$\frac{x^2}{3\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}-\frac{\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{3\,c\left(c^4+\frac{1}{x^4}\right)x\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}$$

Result (type 4, 107 leaves):

$$\frac{1}{3\left(c^2 x^2\right)^{3/2}}x^2\sqrt{\frac{c^2 x^2}{2+2\,c^4 x^4}}$$
$$\left(\sqrt{c^2 x^2}\left(1+c^4 x^4\right)-2\,(-1)^{1/4}\sqrt{1+c^4 x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2 x^2}\right],-1\right]\right)$$

Problem 166: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}{x^3}\,dx$$

Optimal (type 4, 137 leaves, 6 steps):

$$-\frac{\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}{c^2+\frac{1}{x^2}}+$$
$$c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticE}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}-$$
$$\frac{1}{2}c\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}$$

Result (type 4, 53 leaves):

$$-c^2\sqrt{\text{Cosh}[2\,\text{Log}[c\,x]]}\left(\sqrt{\text{Cosh}[2\,\text{Log}[c\,x]]}+i\,\text{EllipticE}[i\,\text{Log}[c\,x],2]\right)\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}$$

Problem 168: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}}{x^5}\,dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{1}{3}\left(c^4+\frac{1}{x^4}\right)\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}+$$
$$\frac{1}{6}c^3\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x\,\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]\sqrt{\text{Sech}[2\,\text{Log}[c\,x]]}$$

Result (type 4, 117 leaves):

$$\frac{1}{3x^4\sqrt{c^2x^2}}\sqrt{2}\sqrt{\frac{c^2x^2}{1+c^4x^4}}$$
$$\left(-\sqrt{c^2x^2}\left(1+c^4x^4\right)+(-1)^{1/4}c^4x^4\sqrt{1+c^4x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right],-1\right]\right)$$

Problem 170: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^7}{\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}\,dx$$

Optimal (type 4, 141 leaves, 7 steps):

$$\frac{4}{77c^4\left(c^4+\frac{1}{x^4}\right)\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{6x^4}{77\left(c^4+\frac{1}{x^4}\right)\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+$$
$$\frac{x^8}{11\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{77c^5\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}$$

Result (type 4, 128 leaves):

$$\left(\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(\sqrt{c^2x^2}\left(4+17c^4x^4+20c^8x^8+7c^{12}x^{12}\right)+\right.\right.$$
$$\left.\left.4(-1)^{1/4}\sqrt{1+c^4x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right],-1\right]\right)\right)\Big/\left(154\sqrt{2}\,c^8\sqrt{c^2x^2}\right)$$

Problem 172: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^5}{\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}\,dx$$

Optimal (type 4, 251 leaves, 9 steps):

$$-\frac{4}{15\,c^4\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{4}{15\,c^4\left(c^4+\frac{1}{x^4}\right)x^2\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{2\,x^2}{15\left(c^4+\frac{1}{x^4}\right)\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{x^6}{9\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{4\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{15\,c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{15\,c^3\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}$$

Result (type 4, 164 leaves):

$$\frac{1}{90\sqrt{2}\,c^6\sqrt{c^2x^2}}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(\left(c^2x^2\right)^{3/2}\left(11+16\,c^4x^4+5\,c^8x^8\right)-12\,(-1)^{3/4}\sqrt{1+c^4x^4}\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right],-1\right]+12\,(-1)^{3/4}\sqrt{1+c^4x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right],-1\right]\right)$$

Problem 174: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3}{\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}\,dx$$

Optimal (type 4, 111 leaves, 6 steps):

$$\frac{2}{7\left(c^4+\frac{1}{x^4}\right)\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{x^4}{7\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}-\frac{2\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{7\,c\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}$$

Result (type 4, 119 leaves):

$$\frac{1}{14\sqrt{2}\,c^4\sqrt{c^2x^2}}\sqrt{\frac{c^2x^2}{1+c^4x^4}}\left(\sqrt{c^2x^2}\left(3+4\,c^4x^4+c^8x^8\right)-4\,(-1)^{1/4}\sqrt{1+c^4x^4}\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[(-1)^{1/4}\sqrt{c^2x^2}\right],-1\right]\right)$$

Problem 176: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x}{\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}\,dx$$

Optimal (type 4, 214 leaves, 8 steps):

$$-\frac{12}{5\left(c^4+\frac{1}{x^4}\right)\left(c^2+\frac{1}{x^2}\right)x^4\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{6}{5\left(c^4+\frac{1}{x^4}\right)x^2\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{x^2}{5\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}+\frac{12\,c\,\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticE}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}-\frac{6\,c\,\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]}{5\left(c^4+\frac{1}{x^4}\right)^2x^3\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}$$

Result (type 4, 171 leaves):

$$\frac{1}{10\,c^2\left(c^2x^2\right)^{3/2}}\sqrt{\frac{c^2x^2}{2+2\,c^4x^4}}\left(\sqrt{c^2x^2}\left(-5-4\,c^4x^4+c^8x^8\right)-12\left(-1\right)^{3/4}c^2x^2\sqrt{1+c^4x^4}\;\text{EllipticE}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{1/4}\sqrt{c^2x^2}\right],-1\right]+12\left(-1\right)^{3/4}c^2x^2\sqrt{1+c^4x^4}\;\text{EllipticF}\left[i\,\text{ArcSinh}\left[\left(-1\right)^{1/4}\sqrt{c^2x^2}\right],-1\right]\right)$$

Problem 180: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}}{x^3}\,dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$\frac{1}{2}\left(c^4+\frac{1}{x^4}\right)x^2\,\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}-\frac{1}{4\,c}\left(c^4+\frac{1}{x^4}\right)\sqrt{\frac{c^4+\frac{1}{x^4}}{\left(c^2+\frac{1}{x^2}\right)^2}}\left(c^2+\frac{1}{x^2}\right)x^3\,\text{EllipticF}\left[2\,\text{ArcCot}[c\,x],\frac{1}{2}\right]\text{Sech}[2\,\text{Log}[c\,x]]^{3/2}$$

Result (type 4, 98 leaves):

$$\frac{1}{\sqrt{c^2 x^2}} \sqrt{2}\ c^2 \sqrt{\frac{c^2 x^2}{1+c^4 x^4}} \left(\sqrt{c^2 x^2} - (-1)^{1/4} \sqrt{1+c^4 x^4}\ \text{EllipticF}\left[i\ \text{ArcSinh}\left[(-1)^{1/4} \sqrt{c^2 x^2}\right], -1\right]\right)$$

Problem 185: Result more than twice size of optimal antiderivative.

$$\int \text{Sech}\left[a + b\, \text{Log}\left[c\, x^n\right]\right]^4 \, dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$\frac{1}{1+4 b n} 16\, e^{4 a}\, x \left(c\, x^n\right)^{4 b} \text{Hypergeometric2F1}\left[4, \frac{1}{2}\left(4+\frac{1}{b n}\right), \frac{1}{2}\left(6+\frac{1}{b n}\right), -e^{2 a} \left(c\, x^n\right)^{2 b}\right]$$

Result (type 5, 750 leaves):

$$\frac{1}{6 b^3 n^3}\left(-1+4 b^2 n^2\right) x\, \text{Sech}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Sech}\left[a+b\, n\, \text{Log}[x]+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Sinh}[b\, n\, \text{Log}[x]] + \frac{1}{3 b n} x\, \text{Sech}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Sech}\left[a+b\, n\, \text{Log}[x]+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]^3 \text{Sinh}[b\, n\, \text{Log}[x]] + \frac{1}{6 b^2 n^2} x\, \text{Sech}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Sech}\left[a+b\, n\, \text{Log}[x]+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]^2 \left(\text{Cosh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]+2 b n\, \text{Sinh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]\right) + \frac{1}{6 b^3 n^3 (1+2 b n)} e^{-\frac{a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]\right)}{b n}} \text{Sech}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \left(e^{\left(2+\frac{1}{b n}\right)\left(a+b\, \text{Log}\left[c x^n\right]\right)} \text{Cosh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Hypergeometric2F1}\left[1, 1+\frac{1}{2 b n}, 2+\frac{1}{2 b n}, -e^{2\left(a+b\, \text{Log}\left[c x^n\right]\right)}\right] - e^{\frac{a}{b n}+\frac{-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]}{n}} (1+2 b n)\, x \left(\text{Cosh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Hypergeometric2F1}\left[1, \frac{1}{2 b n}, 1+\frac{1}{2 b n}, -e^{2\left(a+b\, n\, \text{Log}[x]+b\left(-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]\right)\right)}\right] + \text{Sinh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]\right)\right) - \frac{1}{3 b n (1+2 b n)} 2\, e^{-\frac{a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]\right)}{b n}} \text{Sech}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \left(e^{\left(2+\frac{1}{b n}\right)\left(a+b\, \text{Log}\left[c x^n\right]\right)} \text{Cosh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Hypergeometric2F1}\left[1, 1+\frac{1}{2 b n}, 2+\frac{1}{2 b n}, -e^{2\left(a+b\, \text{Log}\left[c x^n\right]\right)}\right] - e^{\frac{a}{b n}+\frac{-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]}{n}} (1+2 b n)\, x \left(\text{Cosh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right] \text{Hypergeometric2F1}\left[1, \frac{1}{2 b n}, 1+\frac{1}{2 b n}, -e^{2\left(a+b\, n\, \text{Log}[x]+b\left(-n\, \text{Log}[x]+\text{Log}\left[c x^n\right]\right)\right)}\right] + \text{Sinh}\left[a+b\left(-n\, \text{Log}[x]+\text{Log}\left[c\, x^n\right]\right)\right]\right)\right)$$

Problem 187: Result more than twice size of optimal antiderivative.

$$\int \text{Sech}\left[a+2\,\text{Log}\left[c\,\sqrt{x}\,\right]\right]^3\,dx$$

Optimal (type 1, 25 leaves, 3 steps):

$$\frac{2\,c^6\,e^{-a}}{\left(c^4+\frac{e^{-2\,a}}{x^2}\right)^2}$$

Result (type 1, 62 leaves):

$$-\left(\left(2\,\left(\text{Cosh}[a]-\text{Sinh}[a]\right)\,\left(2\,c^4\,x^2+\text{Cosh}[a]^2-2\,\text{Cosh}[a]\,\text{Sinh}[a]+\text{Sinh}[a]^2\right)\right)\Big/\left(c^2\,\left(\left(1+c^4\,x^2\right)\,\text{Cosh}[a]+\left(-1+c^4\,x^2\right)\,\text{Sinh}[a]\right)^2\right)\right)$$

Problem 188: Result more than twice size of optimal antiderivative.

$$\int \text{Sech}\left[a+2\,\text{Log}\left[\frac{c}{\sqrt{x}}\right]\right]^3\,dx$$

Optimal (type 1, 25 leaves, 4 steps):

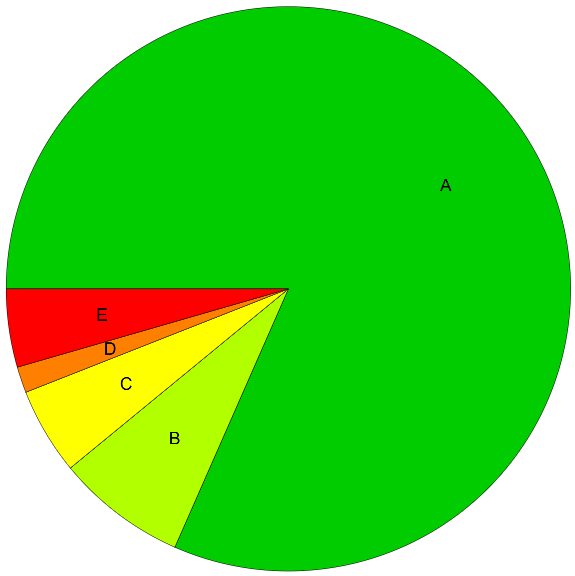
$$\frac{2\,c^2\,e^{-3\,a}}{\left(e^{-2\,a}+\frac{c^4}{x^2}\right)^2}$$

Result (type 1, 64 leaves):

$$-\left(\left(2\,c^6\,\left(\left(c^4+2\,x^2\right)\,\text{Cosh}[a]+\left(c^4-2\,x^2\right)\,\text{Sinh}[a]\right)\,\left(\text{Cosh}[2\,a]+\text{Sinh}[2\,a]\right)\right)\Big/\left(\left(c^4+x^2\right)\,\text{Cosh}[a]+\left(c^4-x^2\right)\,\text{Sinh}[a]\right)^2\right)$$

Summary of Integration Test Results

201 integration problems



A - 164 optimal antiderivatives

B - 15 more than twice size of optimal antiderivatives

C - 10 unnecessarily complex antiderivatives

D - 3 unable to integrate problems

E - 9 integration timeouts